Solutions

Exam III
Chapter 7

Answer the following questions. *You must show your work to receive full credit.* Be sure to make reasonable simplifications. If your answer includes Permutations or Combinations, please find the number it represents. For instance, $C(3,1) = 3$. Indicate your final answer with a $\boxed{\text{box}}$.

1. (6 points) Suppose you roll a die 10 times and your outcomes are 1, 2, 2, 6, 4, 4, 5, 5, 2, 2. Find the relative frequency distribution for your experiment.

Outcome	1	2	3	4	5	6
Rel. Freq.	1/10	4/10	0	2/10	2/10	1/10

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2. (2 points each) Determine which (if any) of the following is a valid probability distribution. For each, *give a brief explanation why.*

(a)

Outcome	6	7	8	9	10
Probability	0	.3	.6	.1	0

Yes, all between 0 and 1 and add to 1.

(b)

Outcome	1	2	3	4	5
Probability	.2	.3	.1	.1	.5

No They add to 1.2.

3.(2 points each) The following table shows the profile, by the math section of the SAT Reasoning Test, of admitted students at UCLA for the Fall 2011 semester.

		7	6	5	4	2	
		700-800	**600-699**	**500-599**	**400-499**	**200-399**	**Total**
A	**Admitted**	6,611	3,981	1,388	385	8	12,373
A'	**Not Admitted**	6,622	13,876	9,890	4,974	1,363	36,725
	Total Applicants	13,233	17,857	11,278	5,359	1,371	49,098

(a) How many students were admitted to UCLA for the Fall 2011 semester?

(b) What is the probability that an applicant was not admitted?

(c) What is the probability that an applicant had a math SAT score of 500 or greater and was not admitted?

(d) What is the probability that an applicant was admitted, given that there math SAT score was between 200 and 399?

(e) Are the events that an applicant was admitted and an applicant has a math SAT score between 400 and 499 independent, mutually exclusive or neither?

(a) $12,373 = n(A)$

(b) $\dfrac{36,725}{49,098} = P(A')$

(c) $\dfrac{6,622 + 13,876 + 9,890}{49,098} = P(A' \cap (7 \cup 6 \cup 5))$

(d) $P(A|2) = \dfrac{n(A \cap 2)}{n(2)} = \dfrac{8}{1,371}$

(e) $P(A \cap 4) = \dfrac{385}{49,098} \neq \dfrac{5,359}{49,098} \cdot \dfrac{12,373}{49,098} = P(A) \cdot P(4)$

not independent, not mutually exclusive,

so neither.

4. (3 points) Let M be the event that a music composer can read music and C the event that a music composer composes classical music. Suppose that 95% of all music composers can read music and that 99% of all classical music composers can read music. Write this given information using conditional expectations.

$$P(\underline{M} \mid \underline{C}) = \underline{.99}$$

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5. (5 points) There is a 20% chance of snow today and a 50% chance of snow tomorrow. Assuming that the event that it snows today is *independent* of the event that it snows tomorrow, what is the probability that it will snow by the end of tomorrow?

A= snow today

B= snow tomorrow

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= .2 + .5 - .2 \cdot .5 = .6.$$

60% chance of snow.

6. (8 points) Professor Lando Calrissian insists that all senior espionage majors take his notorious aptitude test. The test is so tough that only 10% of those *not* going on to a career in espionage (a.k.a. lying to your friends) will pass the test, whereas 60% of the seniors who do go on to a career in espionage still fail the test. Further, 80% of all senior espionage majors will go on to a career in espionage. Assuming that young Boba Fett passes the test, what is the probability that he will continue on to a career in espionage?

C = career in espionage

F = fail test

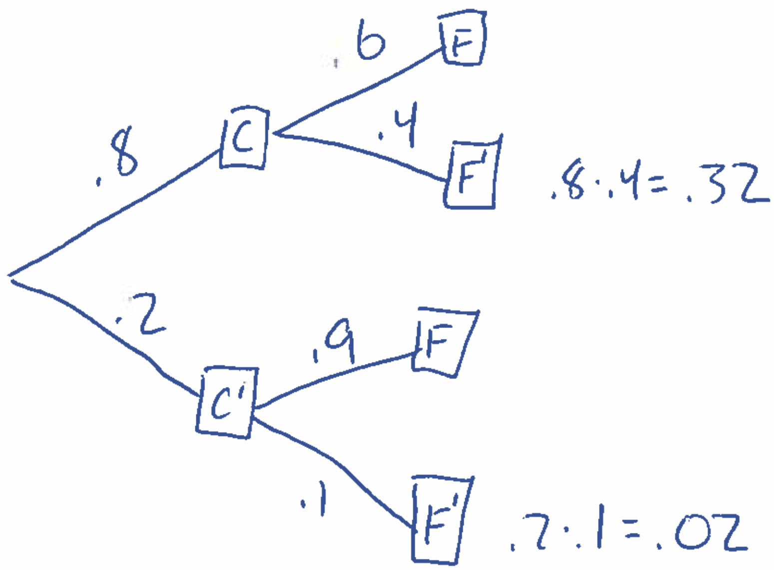
Given

$P(F'|C') = .1$

$P(F|C) = .6$

$P(C) = .8$

Want: $P(C|F')$



$$P(C|F') = \frac{.32}{.32+.02}$$

10 marbles

7. A bag contains two red marbles, three green ones, one pink one, and four orange ones. Assuming that you grab four at random, find the following information.

(a) (1 point) How many different combinations of marbles can you get?

(b) (3 points) How many different ways can you draw at least one marble of each color?

(c) (3 points) What is the probability that you get all the red ones given that you get at least two of the green ones?

(a) $C(10,4) = 210$

(b) Red Green Pink Orange

$$C(2,1) \cdot C(3,1) \cdot C(1,1) \cdot C(4,1) = 24$$

(c) $$P(\text{all reds} \mid \text{at least 2 greens}) = \frac{P(\text{all reds} \cap \geq 2 \text{ greens})}{P(\geq 2 \text{ greens})}$$

$= \cancel{P(\text{at } 2 \text{ reds}}$

$$= \frac{P(\text{all reds} \cap 2 \text{ greens})}{P(\geq 2 \text{ greens})} = \frac{n(\text{all reds} \cap 2 \text{ greens})}{n(\geq 2 \text{ greens})}$$

$$= \frac{C(3,2) \cdot C(2,2)}{C(3,2) \cdot C(7,2) + C(3,3) \cdot C(7,1)} = \frac{3}{63+7} = \frac{3}{70}$$

8. There are three grocery stores in your area: call them A, B, and C. Of those using grocery store A, 10% will continue using A and 10% will switch to B by the end of the year. Of those using grocery store B, 50% will switch to A and 20% will continue using B at the end of the year. Of those using C, 20% will switch to B and 50% will continue using C at the end of the year. Assume that each year, the choice of whether to switch stores is independent of the past.

(a) (3 points) Find the transition matrix P which represents this Markov system.

(b) (2 points) Assuming that twice as many people currently use grocery store B as either one of the others, find the initial distribution vector.

(c) (2 points) What percentage of people will be using grocery store B in five years?

(a)

$$P = \begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{pmatrix} .10 & .10 & .80 \\ .50 & .20 & .30 \\ .30 & .20 & .50 \end{pmatrix} \end{array}$$

(b) $P(B) = 2P(A) = 2P(C)$ $\quad x + 2x + x = 1 \Rightarrow x = \frac{1}{4}$

$$v = \left(\frac{1}{4} \quad \frac{1}{2} \quad \frac{1}{4} \right)$$

(c) $vP^5 = (.27866 \quad .17213 \quad .54921)$

About 17.213% use grocery store B